



Physics Formula Sheet

Natural Constants

Speed of light	$c = 2.998 \times 10^8 \text{ m/s}$	Planck constant	$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$
Reduced Planck $\hbar = h/2\pi$	$\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$	Boltzmann constant	$k_B = 1.381 \times 10^{-23} \text{ J/K}$
Avogadro number	$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$	Gas constant $R = N_A k_B$	$R = 8.314 \text{ J/(mol}\cdot\text{K)}$
Elementary charge	$e = 1.602 \times 10^{-19} \text{ C}$	Electron mass	$m_e = 9.109 \times 10^{-31} \text{ kg}$
Proton mass	$m_p = 1.6726 \times 10^{-27} \text{ kg}$	Neutron mass	$m_n = 1.6749 \times 10^{-27} \text{ kg}$
Gravitational constant	$G = 6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$	Standard gravity	$g = 9.807 \text{ m/s}^2$
Vacuum permittivity	$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$	Vacuum permeability	$\mu_0 = 1.257 \times 10^{-6} \text{ H/m}$
Coulomb constant $k_e = 1/4\pi\epsilon_0$	$k_e = 8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$	Stefan-Boltzmann	$\sigma = 5.671 \times 10^{-8} \text{ W/(m}^2\text{K}^4)$
Bohr radius	$a_0 = 5.292 \times 10^{-11} \text{ m}$	Rydberg energy	$E_R = 13.606 \text{ eV}$
Fine-structure constant	$\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137.036$	Wien constant	$b = 2.898 \times 10^{-3} \text{ m}\cdot\text{K}$
Atomic mass unit	$u = 1.661 \times 10^{-27} \text{ kg}$	Proton-electron mass ratio	$m_p/m_e = 1836.15$

1 Mechanics

1.1 Kinematics

$$\mathbf{v} = \dot{\mathbf{r}}, \quad \mathbf{a} = \dot{\mathbf{v}}$$
$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a\Delta x$$

$\mathbf{r}$ position [m], $\mathbf{v}$ velocity [m/s], $\mathbf{a}$ acceleration [m/s²]

1.2 Dynamics & Conservation

$$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}, \quad \mathbf{p} = m\mathbf{v}$$
$$\mathbf{J} = \Delta\mathbf{p} = \int \mathbf{F} dt \quad (\text{impulse})$$
$$\mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}, \quad U_{\text{grav}} = -\frac{GMm}{r}$$

$\mathbf{p}$ momentum [kg·m/s], $\mathbf{J}$ impulse [N·s], G gravitational constant

Friction

$$f_s \leq \mu_s N, \quad f_k = \mu_k N$$

μ_s, μ_k static/kinetic friction coefficients, N normal force

1.3 Work, Energy & Power

$$W = \int \mathbf{F} \cdot d\mathbf{s}$$
$$E_k = \frac{1}{2}mv^2, \quad U_{\text{spring}} = \frac{1}{2}kx^2$$
$$E_k + U = \text{const} \quad (\text{conservative forces})$$
$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

W work [J], k spring constant [N/m], P power [W]

1.4 Rotational Mechanics

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}, \quad \boldsymbol{\tau} = \dot{\mathbf{L}}$$
$$E_{\text{rot}} = \frac{1}{2}I\omega^2$$
$$I = \int r^2 dm, \quad I = I_{\text{cm}} + md^2 \quad (\text{parallel axis})$$

$\boldsymbol{\tau}$ torque [N·m], I moment of inertia [kg·m²], $\boldsymbol{\omega}$ angular velocity [rad/s],
 $\mathbf{L}$ angular momentum [kg·m²/s]

Common Moments of Inertia

Solid sphere	$\frac{2}{5}mR^2$	Hollow sphere	$\frac{2}{3}mR^2$
Solid cylinder	$\frac{1}{2}mR^2$	Thin ring	mR^2
Rod (centre)	$\frac{1}{12}mL^2$	Rod (end)	$\frac{1}{3}mL^2$

1.5 Oscillations

$$\ddot{x} + \omega_0^2 x = 0 \Rightarrow x(t) = A \cos(\omega_0 t + \varphi)$$
$$\omega_0 = \sqrt{k/m} \quad (\text{spring}), \quad \omega_0 = \sqrt{g/L} \quad (\text{pendulum})$$
$$T = 2\pi/\omega_0, \quad f = \omega_0/2\pi$$

ω_0 natural frequency [rad/s], A amplitude, φ initial phase, T period [s]

Damped & Forced Oscillation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \omega t$$
$$\omega' = \sqrt{\omega_0^2 - \gamma^2}, \quad x_{\text{damp}} = Ae^{-\gamma t} \cos(\omega' t + \varphi)$$
$$A_{\text{res}} = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}}$$

γ damping coefficient, ω' damped frequency, A_{res} forced amplitude

2 Electrodynamics

2.1 Electrostatics

$$\begin{aligned}\mathbf{F} &= \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r} \quad (\text{Coulomb}) \\ \mathbf{E} &= -\nabla V, \quad V = \frac{q}{4\pi\epsilon_0 r} \\ \oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enc}}}{\epsilon_0} \Leftrightarrow \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{Gauss}) \\ U &= \frac{q_1 q_2}{4\pi\epsilon_0 r}, \quad u_E = \frac{1}{2}\epsilon_0 E^2\end{aligned}$$

$\mathbf{E}$ electric field [V/m], V potential [V], ρ charge density [C/m³], u_E energy density [J/m³]

Capacitance

$$C = Q/V, \quad U = Q^2/2C = \frac{1}{2}CV^2, \quad C_{\text{pp}} = \epsilon_0 A/d$$

C capacitance [F], A plate area [m²], d separation [m]

Dielectrics

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_r \epsilon_0 \mathbf{E}, \quad \nabla \cdot \mathbf{D} = \rho_{\text{free}}$$

$\mathbf{D}$ displacement field [C/m²], $\mathbf{P}$ polarisation, ϵ_r relative permittivity

2.2 Magnetostatics

$$\begin{aligned}d\mathbf{B} &= \frac{\mu_0 I}{4\pi} \frac{d\mathbf{l} \times \hat{r}}{r^2} \quad (\text{Biot-Savart}) \\ \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I_{\text{enc}} \Leftrightarrow \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (\text{Ampère}) \\ \mathbf{F} &= q\mathbf{v} \times \mathbf{B} = I\mathbf{L} \times \mathbf{B} \quad (\text{Lorentz / Laplace})\end{aligned}$$

$\mathbf{B}$ magnetic flux density [T], I current [A]

Useful Field Results

Long wire	$B = \mu_0 I / 2\pi r$
Solenoid (n turns/m)	$B = \mu_0 n I$
Magnetic dipole moment	$\boldsymbol{\mu} = I\mathbf{A}, \boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

Inductance & Magnetic Energy

$$\mathcal{E} = -L\dot{I}, \quad U = \frac{1}{2}LI^2, \quad u_B = B^2/2\mu_0$$

L inductance [H], $\mathcal{E}$ EMF [V], u_B energy density [J/m³]

2.3 Maxwell's Equations (SI)

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho/\epsilon_0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\partial\mathbf{B}/\partial t \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial\mathbf{E}/\partial t\end{aligned}$$

$\mathbf{J}$ current density [A/m²]

2.4 Circuits

$$\begin{aligned}V &= IR, \quad P = IV = I^2 R = V^2/R \\ R &= \rho l/A, \quad \rho = \rho_0[1 + \alpha(T - T_0)] \\ Z_R &= R, \quad Z_C = 1/i\omega C, \quad Z_L = i\omega L \\ \tau_{RC} &= RC, \quad \tau_{RL} = L/R, \quad \omega_0 = 1/\sqrt{LC}\end{aligned}$$

ρ resistivity [$\Omega \cdot \text{m}$], l length, A cross-section, Z impedance [Ω], τ time constant

2.5 Electromagnetic Waves

$$\begin{aligned}c &= 1/\sqrt{\mu_0 \epsilon_0}, \quad E_0 = cB_0 \\ \mathbf{S} &= \mu_0^{-1} \mathbf{E} \times \mathbf{B} \quad (\text{Poynting vector}) \\ \langle S \rangle &= \epsilon_0 c E_0^2/2 = cB_0^2/2\mu_0 \quad (\text{intensity}) \\ P_{\text{rad}} &= \langle S \rangle/c \quad (\text{absorbing}), \quad P_{\text{rad}} = 2\langle S \rangle/c \quad (\text{reflecting})\end{aligned}$$

$\mathbf{S}$ Poynting vector [W/m²], E_0, B_0 amplitudes

3 Thermodynamics

3.1 Laws of Thermodynamics

$$\begin{aligned}dU &= \delta Q - \delta W, \quad \delta W = p dV \quad (\text{1st}) \\ dS &\geq \delta Q/T \quad (\text{2nd, Clausius}), \quad S \rightarrow 0 \text{ as } T \rightarrow 0 \quad (\text{3rd})\end{aligned}$$

U internal energy [J], Q heat [J], W work [J], S entropy [J/K], T temperature [K]

3.2 Ideal Gas

$$\begin{aligned}pV &= nRT = Nk_B T \\ U &= \frac{f}{2} Nk_B T \quad (f \text{ degrees of freedom}) \\ C_V &= \frac{f}{2} R, \quad C_p = C_V + R, \quad \gamma = C_p/C_V\end{aligned}$$

n moles, N molecules, f : 3 (monatomic), 5 (diatomic, rigid), γ adiabatic index

Thermodynamic Processes

Isothermal	$T = \text{const}$	$W = nRT \ln(V_2/V_1)$
Isochoric	$V = \text{const}$	$W = 0, \Delta U = Q$
Isobaric	$p = \text{const}$	$W = p\Delta V$
Adiabatic	$Q = 0$	$pV^\gamma = \text{const}, TV^{\gamma-1} = \text{const}$

Carnot Efficiency

$$\eta_{\text{Carnot}} = 1 - T_C/T_H, \quad \text{COP}_{\text{refrigerator}} = T_C/(T_H - T_C)$$

T_C cold, T_H hot reservoir temperature [K]

3.3 Thermodynamic Potentials

$$\begin{aligned}H &= U + pV \quad (\text{enthalpy}) \\ F &= U - TS \quad (\text{Helmholtz free energy}) \\ G &= H - TS = U - TS + pV \quad (\text{Gibbs free energy}) \\ dG &= -S dT + V dp + \mu dN\end{aligned}$$

μ chemical potential [J/mol], H enthalpy [J]; equilibrium: $dG = 0$ at const T, p

3.4 Statistical Mechanics

$$\begin{aligned}S &= k_B \ln \Omega \quad (\text{Boltzmann entropy}) \\ Z &= \sum_i e^{-E_i/k_B T}, \quad \beta = 1/k_B T \\ \langle E \rangle &= -\partial \ln Z / \partial \beta, \quad F = -k_B T \ln Z\end{aligned}$$

Ω number of microstates, Z canonical partition function

Maxwell-Boltzmann Speed Distribution

$$\begin{aligned}f(v) &= 4\pi n \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-mv^2/2k_B T} \\ v_{\text{mp}} &= \sqrt{2k_B T/m}, \quad \langle v \rangle = \sqrt{8k_B T/\pi m}, \quad v_{\text{rms}} = \sqrt{3k_B T/m}\end{aligned}$$

Heat Transfer

$$\begin{aligned}\dot{Q}_{\text{cond}} &= -\kappa A \nabla T \text{ (conduction)} \\ \dot{Q}_{\text{rad}} &= \sigma A T^4 \text{ (black body)}, \quad \dot{Q}_{\text{conv}} = h A \Delta T\end{aligned}$$

κ thermal conductivity [W/(m·K)], h convective coefficient [W/(m²K)]

4 Quantum Mechanics

4.1 Wave–Particle Duality

$$\begin{aligned}E &= hf = \hbar\omega, \quad p = h/\lambda = \hbar k \\ \lambda_{\text{dB}} &= h/p \quad (\text{de Broglie wavelength})\end{aligned}$$

f frequency [Hz], λ wavelength [m], $k = 2\pi/\lambda$ wave number [m^{−1}]

4.2 Schrödinger Equation

$$\begin{aligned}i\hbar \frac{\partial \Psi}{\partial t} &= \hat{H} \Psi \quad (\text{time-dependent}) \\ \hat{H} \psi &= E \psi \quad (\text{time-independent}) \\ \hat{H} &= -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})\end{aligned}$$

Ψ wave function, $\hat{H}$ Hamiltonian operator, $V(\mathbf{r})$ potential energy

Normalisation & Expectation Values

$$\int_{-\infty}^{\infty} |\Psi|^2 dx = 1, \quad \langle \hat{A} \rangle = \int \Psi^* \hat{A} \Psi dx$$

Common 1D Solutions

Particle in box ($0 < x < L$) $E_n = n^2 \pi^2 \hbar^2 / 2mL^2$, $n = 1, 2, \dots$
Harmonic oscillator $E_n = \hbar \omega_0 (n + \frac{1}{2})$, $n = 0, 1, 2, \dots$

4.3 Uncertainty Principle

$$\Delta x \Delta p \geq \hbar/2, \quad \Delta E \Delta t \geq \hbar/2$$

Δ standard deviation of observable

4.4 Operators

$$\begin{aligned}\hat{x} &= x, \quad \hat{p} = -i\hbar \nabla, \quad \hat{E} = i\hbar \partial_t \\ [\hat{x}, \hat{p}] &= i\hbar, \quad [\hat{L}_i, \hat{L}_j] = i\hbar \varepsilon_{ijk} \hat{L}_k\end{aligned}$$

4.5 Angular Momentum

$$\begin{aligned}\hat{L}^2 Y_l^m &= \hbar^2 l(l+1) Y_l^m, \quad \hat{L}_z Y_l^m = \hbar m Y_l^m \\ l &= 0, 1, 2, \dots; \quad m = -l, \dots, +l \\ \hat{S}^2 |s, m_s\rangle &= \hbar^2 s(s+1) |s, m_s\rangle, \quad \hat{S}_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle\end{aligned}$$

Y_l^m spherical harmonics, l orbital q.n., m magnetic q.n., s spin q.n.

4.6 Hydrogen Atom

$$\begin{aligned}E_n &= -\frac{E_R}{n^2} = -\frac{13.606 \text{ eV}}{n^2}, \quad n = 1, 2, 3, \dots \\ a_0 &= \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 5.292 \times 10^{-11} \text{ m}\end{aligned}$$

n principal q.n., $E_R = 13.606 \text{ eV}$ Rydberg energy, a_0 Bohr radius

Spectral Series & Selection Rules

$$\begin{aligned}\frac{1}{\lambda} &= R_\infty \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad R_\infty = 1.097 \times 10^7 \text{ m}^{-1} \\ \Delta l &= \pm 1, \quad \Delta m = 0, \pm 1, \quad \Delta s = 0 \text{ (E1 rules)}\end{aligned}$$

5 Special Relativity

5.1 Lorentz Transformation (S' moves at v along x)

$$\begin{aligned}t' &= \gamma(t - vx/c^2), \quad x' = \gamma(x - vt) \\ \gamma &= 1/\sqrt{1 - \beta^2}, \quad \beta = v/c \\ \Delta t &= \gamma \Delta t_0 \text{ (time dilation)}, \quad L = L_0/\gamma \text{ (length contraction)}\end{aligned}$$

γ Lorentz factor, Δt_0 proper time, L_0 proper length

Relativistic Velocity Addition

$$u' = \frac{u - v}{1 - uv/c^2}$$

5.2 Relativistic Energy & Momentum

$$\begin{aligned}E &= \gamma mc^2, \quad E_0 = mc^2, \quad E_k = (\gamma - 1)mc^2 \\ \mathbf{p} &= \gamma m \mathbf{v} \\ E^2 &= (pc)^2 + (mc^2)^2 \quad (\text{dispersion relation})\end{aligned}$$

E_0 rest energy, E_k kinetic energy, m invariant rest mass

5.3 Spacetime Interval & Doppler

$$\begin{aligned}s^2 &= -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \text{ (invariant interval)} \\ f_{\text{obs}} &= f_{\text{src}} \sqrt{\frac{1 \mp \beta}{1 \pm \beta}} \quad (-/+ : \text{receding/approaching})\end{aligned}$$

6 Fluid Dynamics

6.1 Fundamental Equations

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \quad (\text{continuity}) \\ A_1 v_1 &= A_2 v_2 \quad (\text{incompressible flow}) \\ p + \frac{1}{2} \rho v^2 + \rho g z &= \text{const} \quad (\text{Bernoulli})\end{aligned}$$

ρ density [kg/m³], $\mathbf{v}$ velocity field [m/s], p pressure [Pa], z height [m]

6.2 Viscous Flow

$$\begin{aligned}\tau_{\text{shear}} &= \eta dv/dy \quad (\text{Newton's viscosity law}) \\ \text{Re} &= \rho v L / \eta \quad (\text{Reynolds number}) \\ Q &= \pi R^4 \Delta p / 8 \eta L \quad (\text{Hagen-Poiseuille})\end{aligned}$$

η dynamic viscosity [Pa·s], $\text{Re} \lesssim 2300$ laminar, $\text{Re} \gtrsim 4000$ turbulent, Q flow rate [m³/s]

Navier–Stokes (incompressible, $\nabla \cdot \mathbf{v} = 0$)

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v}) = -\nabla p + \eta \nabla^2 \mathbf{v} + \mathbf{f}$$

$\mathbf{f}$ body force per unit volume

Drag, Buoyancy & Surface Tension

$$F_{\text{Stokes}} = 6\pi\eta r v \text{ (sphere)}, \quad F_b = \rho_f V_{\text{sub}} g$$
$$\Delta p = 2\gamma/R \text{ (bubble)}, \quad \Delta p = 4\gamma/R \text{ (soap film)}$$

r sphere radius, γ surface tension [N/m], R radius of curvature

7 Optics

7.1 Geometric Optics

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (\text{Snell's law})$$
$$\theta_B = \arctan(n_2/n_1) \quad (\text{Brewster's angle})$$
$$\sin \theta_c = n_2/n_1 \quad (\text{critical angle, TIR for } n_1 > n_2)$$

n refractive index, θ angle from surface normal

Thin Lens & Mirror Equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}, \quad m = -d_i/d_o$$
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (\text{lensmaker's equation})$$

f focal length, d_o object distance, d_i image distance (+: real, -: virtual), m magnification

7.2 Wave Optics

Double-Slit Interference (Young)

$$d \sin \theta = m\lambda \text{ (max)}, \quad d \sin \theta = (m + \frac{1}{2})\lambda \text{ (min)}$$
$$\Delta y = \lambda L/d \quad (\text{fringe spacing})$$

d slit separation, λ wavelength, $m \in \mathbb{Z}$, L screen distance

Single-Slit Diffraction

$$a \sin \theta = m\lambda \text{ (minima)}, \quad I(\theta) = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2, \quad \alpha = \frac{\pi a \sin \theta}{\lambda}$$

a slit width, first minimum at $\sin \theta = \lambda/a$

Diffraction Grating & Resolving Power

$$d \sin \theta = m\lambda, \quad \mathcal{R} = \lambda/\Delta\lambda = mN$$

d grating spacing, N total number of slits, $\mathcal{R}$ resolving power

Thin Film Interference

$$2nt = m\lambda \text{ (constructive, no net phase shift)}$$
$$2nt = (m + \frac{1}{2})\lambda \text{ (destructive)}$$

n film index, t thickness; reflection from denser medium adds $\lambda/2$ phase shift

Photons

$$c = f\lambda, \quad E_{\text{photon}} = hf = hc/\lambda, \quad p_{\text{photon}} = h/\lambda = E/c$$

8 Particle Physics

8.1 Radioactive Decay

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$
$$\lambda = \ln 2/t_{1/2}, \quad \tau = 1/\lambda, \quad A = \lambda N$$

λ decay constant [s^{-1}], τ mean lifetime [s], $t_{1/2}$ half-life [s], A activity [Bq]

Q-Value of a Reaction

$$Q = (m_i - m_f)c^2 = K_f - K_i$$

$Q > 0$: exothermic/exoergic; $Q < 0$: endothermic, requires threshold energy

8.2 Standard Model Particles

Quarks ($\times 3$ colours)	u, d ($Q = +\frac{2}{3}, -\frac{1}{3}$), c, s, t, b
Leptons	$e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau$ (+ antiparticles)
Gauge bosons	γ (EM), $W^\pm, Z^0$ (weak), g gluons (strong, 8)
Higgs	$H^0, m_H \approx 125 \text{ GeV}/c^2$

9 Solid State Physics

9.1 Crystal Structure & Diffraction

$$2d \sin \theta = m\lambda \quad (\text{Bragg's law})$$
$$\mathbf{G} \cdot \mathbf{R} = 2\pi n \quad (\text{reciprocal lattice condition})$$

d plane spacing, θ Bragg angle, $\mathbf{G}$ reciprocal lattice vector, $\mathbf{R}$ lattice vector, $n \in \mathbb{Z}$

9.2 Free Electron Model (Drude)

$$\sigma = ne^2\tau/m_e, \quad \mathbf{J} = \sigma \mathbf{E}$$
$$\mu_e = e\tau/m_e \quad (\text{mobility})$$

σ conductivity [$\Omega^{-1}\text{m}^{-1}$], τ scattering time, n carrier density [m^{-3}], μ_e mobility [$\text{m}^2/(\text{V}\cdot\text{s})$]

Fermi Energy (3D, $T=0$)

$$E_F = \frac{\hbar^2}{2m_e} (3\pi^2 n)^{2/3}, \quad g(E) = \frac{3n}{2E_F} \left(\frac{E}{E_F} \right)^{1/2}$$

E_F Fermi energy, $g(E)$ density of states [$\text{J}^{-1}\text{m}^{-3}$]

Hall Effect

$$R_H = V_H/(IB) = -1/(ne) \quad (R_H > 0 \text{ for holes})$$

V_H Hall voltage, sign determines carrier type

9.3 Semiconductors

$$n_i^2 = np = N_C N_V e^{-E_g/k_B T}$$
$$I = I_0(e^{eV/k_B T} - 1) \quad (\text{Shockley diode equation})$$

n_i intrinsic carrier density, E_g band gap, N_C, N_V effective DOS at band edges

9.4 Phonons & Lattice Specific Heat

$$C_V^{\text{Debye}} \propto T^3 \text{ (low } T), \quad C_V \rightarrow 3Nk_B \text{ (Dulong-Petit, high } T)$$
$$E = \hbar\omega_k \quad (\text{phonon energy})$$

θ_D Debye temperature; Dulong-Petit: classical limit $3R \approx 24.9 \text{ J}/(\text{mol}\cdot\text{K})$

9.5 Superconductivity

$R = 0$ for $T < T_c$ (zero resistance)
 $B = 0$ inside superconductor (Meissner effect)

T_c critical temperature; above T_c or critical field B_c : superconductivity breaks down

10 Plasma Physics

10.1 Plasma Parameters

$\lambda_D = \sqrt{\epsilon_0 k_B T / n e^2}$ (Debye shielding length)
 $\omega_p = \sqrt{n e^2 / \epsilon_0 m_e}$ (electron plasma frequency)
 $\omega_{c,e} = e B / m_e$ (electron cyclotron frequency)
 $r_L = m v_\perp / e B$ (Larmor / gyroradius)

n plasma density [m^{-3}], λ_D Debye length; plasma condition: $n \lambda_D^3 \gg 1$

10.2 Plasma Waves & Cutoffs

$\omega^2 = \omega_p^2 + k^2 c^2$ (EM wave in unmagnetised plasma)
 $n_{\text{refr}} = \sqrt{1 - \omega_p^2 / \omega^2}$ (refractive index)

Wave evanescent (n_{refr} imaginary) when $\omega < \omega_p$; cutoff at $\omega = \omega_p$

11 Astrophysics

11.1 Orbital Mechanics & Kepler

$T^2 = \frac{4\pi^2}{GM} a^3$ (Kepler's 3rd law)
 $v_{\text{orb}} = \sqrt{GM/r}$, $v_{\text{esc}} = \sqrt{2GM/R}$
 $E_{\text{orbit}} = -GMm/2a$ (total mechanical energy)

T period, a semi-major axis, r orbital radius, R body radius, v_{esc} escape

speed

11.2 Stellar Physics

$L = 4\pi R_\star^2 \sigma T_{\text{eff}}^4$ (luminosity)
 $\lambda_{\text{max}} T = b = 2.898 \times 10^{-3} \text{ m}\cdot\text{K}$ (Wien's law)
 $F = L/4\pi d^2$ (flux from source at distance d)
 $m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)$ (Pogson's law)

L luminosity [W], $R_\star$ stellar radius, T_{eff} effective temperature, F flux [W/m^2], m apparent magnitude

Distance Modulus

$\mu = m - M = 5 \log_{10}(d/10 \text{ pc})$

M absolute magnitude, d distance in parsecs; $1 \text{ pc} = 3.086 \times 10^{16} \text{ m}$

11.3 Black Holes & General Relativity

$ds^2 = -f c^2 dt^2 + f^{-1} dr^2 + r^2 d\Omega^2$
 $f = 1 - r_s/r$, $r_s = 2GM/c^2$ (event horizon)
 $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ (Einstein field equations)

r_s Schwarzschild radius, Λ cosmological constant, $T_{\mu\nu}$ stress-energy tensor, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$

Gravitational Redshift & Time Dilation

$z_{\text{grav}} \approx \Delta\Phi/c^2 = GM/Rc^2$, $\Delta t_{\text{far}} = \Delta t_{\text{near}}/\sqrt{1 - r_s/r}$

11.4 Cosmology

$v = H_0 d$ (Hubble–Lemaître law), $H_0 \approx 70 \text{ km/s/Mpc}$
 $H = \dot{a}/a$ (Hubble parameter), $z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$

H_0 Hubble constant, $a(t)$ cosmic scale factor, z cosmological redshift